S–5296

Sub. Code
22MMA1C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

First Semester

Mathematics

ALGEBRA – I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an automorphism of a group.

2. Define permutation group.

3. Define external direct product of groups.

4. Define normalizer of an element in G.

5. Define a ring homomorphism.

6. Define zero-divisor of a ring.

7. When will you say that a ring R is said to be imbedded in a ring R' ?

8. If I is an ideal of a ring R containing the unit element, show that I = R.

9. Define unique factorization domain.

10. Define relatively prime element.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let $\theta : G \to H$ be a onto group homomorphism with kernel K. Prove that G/K is isomorphic to H.

Or

(b) State and prove the Cauchy's theorem for Abelian groups.

12. (a) Show that any two p-sylow subgroups of a group G are conjugate.

Or

(b) Suppose that G is the internal direct product of $N_{1,} N_{2,} .....N_n$ Prove that for $i \neq j, N_i \cap N_i = \{e\}$ and if $a \in N_i, b \in N_j$ then $ab = ba$.

13. (a) Define a field. Prove that any field is an integral domain.

Or

(b) Prove that a ring homomorphism $\phi : R \to R'$ is one to one if and only if the Kernel of ϕ is zero sub module.

14. (a) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. The prove that R is a field.

Or

(b) Prove that the mapping $\phi : D \to F$ defined by $\phi(a) = [a,1]$ is an isomorphism of D into F.

15. (a) If R is an Euclidean domain then prove that any two elements a and b in R have a greatest common divisor d.

Or

(b) State and prove the Unique factorization theorem.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. State and prove the Cayley's theorem.

17. State and prove third part of sylow's theorem.

18. lf ϕ is a homomorphism of R into R', then prove the following.

(a) $\phi(O) = O$;

(b) $\phi(-a) = \phi(a)$ for every $a \in R$

19. If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

20. (a) State and prove the division algorithm

(b) State and prove the Gauss lemma.

S–5300

Sub. Code
22MMA2C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Second Semester

Mathematics

ALGEBRA – II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define subspace of a vector space. Give an example.

2. In a vector space show that $\alpha(v - w) = \alpha v - \alpha w$.

3. Define the annihilator of W.

4. Define an orthonormal set.

5. What is meant by algebraic extension of F?

6. If $a \in K$ is a roof of $p(x) \in F[x]$, where $F \subset K$, then prove that in $k[x]$, $(x - a) | p(x)$.

7. Define range of T.

8. When will you say that a matrix is said to be diagonal matrix?

9. When will you say that the linear transformation is said to be unitary?

10. Define the following terms:

(a) Hermitian;

(b) Normal.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) If V is the internal direct sum of $U_1, U_2, ... U_n$, then prove that V is isomorphic to the external direct sum of $U_1, U_2, ..., U_n$.

Or

(b) If $v_1, v_2, ... v_n \in V$ are linearly independent, then prove that every element in their linear span has a unique representation in the form $\lambda_1 u_1 + \lambda_2 u_2 + ..., \lambda_n u_n$ with the $\lambda_i \in F$.

12. (a) With the usual notations, prove that $A(A(W)) = W$.

Or

(b) State and prove the Schwarz inequality.

13. (a) State and prove the Remainder theorem.

Or

(b) If $f(x) \in F[x]$ is irreducible, then prove the following :

(i) If the characteristic of F is 0, $f(x)$ has no multiple roots.

(ii) If the characteristic of F is $p \neq 0$, $f(x)$ has a multiple root only if it is of the form $f(x) = g(x^p)$.

14. (a) If V if finite - dimensional over F, then prove that $S, T \in A(V)$.

(i) $r(ST) \leq r(T)$;

(ii) $r(TS) \leq r(T)$;

(iii) $r(ST) = r(TS) = r(T)$, for S regular in $A(V)$.

Or

(b) If $\lambda \in F$ is characteristic root of $T \in A(V)$, then prove that for any polynomial $q(x) \in F(x)$, $q(\lambda)$ is a characteristic root of $q(T)$.

15. (a) If $T \in A(V)$ is such that $(vT, v) = 0$ for all $v \in V$, then prove that $T = 0$.

Or

(b) (i) If $T \in A(V)$ is Hermitian, then prove that all its characteristic roots are real.

(ii) If N is normal and if $vN^k = 0$, then prove that $vN = 0$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If $v_1, v_2, ..., v_n$ is a basis of V over F and if $w_1, w_2, ,...w_m$ in V are linearly independent of F, then prove that $m \leq n$.

17. (a) State and prove the Bessel inequality.

(b) In V prove that parallelogram law:

$$\|u + v\|^2 + \|u - v\|^2 = 2\left(\|u\|^2 + \|v\|^2\right).$$

18. If F is of characteristics O and if a, b are algebraic over F, then prove that there exists an element $c \in F(a,b)$ such that $F(a,b) = F(c)$.

19. Let $V = F^{(3)}$ and suppose that $\begin{pmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$ is the matrix of $T \in A(V)$ in the basis $v_1 = (1,0,0), v_2 = (0,1,0), v_3 = (0,0,1)$. Find the matrix of T in the basis $u_1(1,1,0), u_2 = (1,2,0)$ and $u_3 = (1,2,1)$.

20. If $T \in A(V)$ has all its characteristic roots in F, then prove that there is a basis of V in which the matrix of T is triangular.

S–5301

Sub. Code
22MMA2C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Second Semester

Mathematics

ANALYSIS – II

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. When will you say that the partition p^* is a refinement of P?

2. Define a rectifiable curve.

3. State the Cauchy criterion for uniform convergence theorem.

4. Define a pointwise bounded sequence functions.

5. Define a power series.

6. When will you say that $\{\phi_n\}$ is said to be orthonormal?

7. If $m^*E = 0$, then prove that E is measurable.

8. Define the characteristic function χ_A.

9. Define the Riemann integrable function.

10. Define a simple function.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) (i) If $f_1(x) \le f_2(x)$ on $[a,b]$, then prove that $\int_a^b f_1 dx \le \int_a^b f_2 dx$.

(ii) If $f \in \mathscr{R}(\alpha)$ on $[a,b]$ and if $|f(x)| \le M$ on $[a,b]$, then prove that $\left|\int_a^b f d\alpha\right| \le M[\alpha(b) - \alpha(a)]$.

Or

(b) If $\vec{f}$ maps $[a,b]$ into R^K and if $\vec{f} \in \mathscr{R}(\alpha)$ for some monotonically increasing function α on $[a,b]$, then prove that $|\vec{f}| \in \mathscr{R}(\alpha)$ and $\left|\int_a^b f d\alpha\right| \le \int |\vec{f}| d\alpha$.

12. (a) State and prove the Cauchy criterion for uniform convergence theorem.

Or

(b) If $\{f_n\}$ is a pointwise bounded sequence of complex functions on a countable set *E,* then prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$.

13. (a) Given a double sequence $\{a_{ij}\}$, $i = 1,2,3,\ldots$, $j = 1,2,3,\ldots$, suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_i (i = 1,2,3,\ldots)$ and $\sum b_i$ converges. Prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} a_{ij}$.

Or

(b) Let e^x be defined on R'. Prove the following:

(i) e^x is continuous and differentiable for all x;

(ii) $(e^x)' = e^x$;

(iii) $e^{x+y} = e^x e^y$;

(iv) $\lim_{x\to\infty} x^n e^{-x} = 0$, for every n.

14. (a) If E_1 and E_2 are measurable, then prove that $m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2$.

Or

(b) Prove that the interval (a, ∞) is measurable.

15. (a) State and prove the Egoroffs theorem.

Or

(b) Define a simple function. Let ϕ be and ψ be simple functions which vanish outside a set of finite measure. Prove that $\int (a\phi + b\psi) = a\int \phi + b\int \psi$. Also prove $\int \phi \geq \int \psi$ if $\phi \geq \psi$ almost everywhere.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If γ' is continuous on $[a,b]$, then prove that γ is rectifiable, and $\wedge(\gamma) = \int_a^b |\gamma'(t)| dt$.

17. State and prove the Stone-Weierstrass theorem.

18. Define the Gamma function. Prove that following:

    (a) The functional equation $\Gamma(x+1) = x\Gamma(x)$ holds if $0 < x < \infty$.

    (b) $\Gamma(n+1) = n!$ for $n = 1,2,3,\ldots$

    (c) $\log\Gamma$ is convex on $(0,\infty)$.

19. Prove that the outer measure of an interval is its length.

20. State and prove the Lusin's theorem.

S–5303

Sub. Code
22MMA2C4

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Second Semester

Mathematics

PROBABILITY AND STATISTICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions

1. Let $f(x) = cx$, $x = 1,2,3,4,5,6$, zero elsewhere be the p.d.f. of x. Find the constant c.

2. Let x have the p.d.f. $f(x) = 2(1-x)$, $0 < x < 1$, zero elsewhere. Find $E\left[6x + 3x^2\right]$.

3. Let $f(x,y) = 6x^2 y$, $0 < x < 1$, $0 < y < 1$, zero elsewhere be the p.d.f of two random variables x and y. Determine $\Pr\left(0 < x < \frac{3}{4}, \frac{1}{3} < y < 2\right)$.

4. Prove that $\Pr(a < x_1 < b, c < x_2 < d) \neq \Pr(a < x_1 < b)$ $\Pr(c < x_2 < d)$ where x_1 and x_2 are dependent random variables with marginal density functions $f(x)$ and $f_2(x)$, respectively.

5. If the m.g.f. of a random variable x is $\left(\frac{1}{3}+\frac{2}{3}e^{t}\right)^{5}$, find $\Pr(x=2 \text{ or } 3)$.

6. Let x be $N(2, 25)$. Find $\Pr(0<x<10)$.

7. Define t-distribution.

8. Let x have the p.d.f. $f(x)=\left(\frac{1}{2}\right)^{x}, x=1,2,3,\ldots,$ zero elsewhere find the p.d.f. or $y=x^{3}$.

9. Define convergence in distribution.

10. Let z_n be $x^2(n)$. Find the mean and variance of z_n.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Find the mean and variance of the p.d.f. $f(x)=6x(1-x), 0<x<1$, zero elesewhere.

Or

(b) Let $f(x)=\frac{x}{6}, x=1,2,3,$ zero elsewhere, be the p.d.f. of x. Find the distribution function and the p.d.f. of $y=x^{2}$.

12. (a) Prove that $E[E(x_2|x_1)]=E(x_2)$ and $Var[E(x_2|x_1)] \leq Var(x_2)$.

Or

(b) Let x_1 and x_2 have the joint p.d.f. $f(x_1,x_2)=15x_1^2x_2$, $0<x_1<x_2<1$, zero elsewhere. Find each marginal p.d.f. and compute $\Pr(x_1+x_2\leq 1)$.

13. (a) Compute the measures of skewness and kurtosis of the binomial distribution $b(n, p)$.

Or

(b) Find the mean and variance of a Poisson distribution.

14. (a) Let x have the uniform distribution over the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Show that $y = \tan x$ has a Cauchy distribution.

Or

(b) Let x_i denote a random variable with mean μ_i and variance $\sigma_i^2, i = 1, 2, \ldots, n$. Find the mean and variance of a linear function $y = k_1 x_1 + K_2 x_2 + \cdots + k_n x_n$ where x_i's are independent random variables and k_i's are constants.

15. (a) Let z_n be $x^2(n)$. Prove that the random variable $y_n = \frac{z_n - n}{\sqrt{2n}}$ has a limiting standard normal distribution.

Or

(b) Let S_n^2 denote the variance of a random sample of size n from a distribution $N(\mu, \sigma^2)$. Prove that $\frac{ns_n^2}{n-1}$ converges in probability to σ^2.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove Chebyshev's inequality.

(b) Find the m.g.f. for the random variable x with p.d.f. $f(x) = \frac{1}{k}$, $x = 1,2,\ldots,k$, zero elsewhere.

17. Let x and y have the joint p.d.f. described as follows

(x,y)	(1,1)	(1,2)	(1,3)	(2,1)	(2,2)	(2,3)
$f(x,y)$	$\frac{2}{15}$	$\frac{4}{15}$	$\frac{3}{15}$	$\frac{1}{15}$	$\frac{1}{15}$	$\frac{4}{15}$

and $f(x,y)$ is equal to zero elsewhere.

(a) Find the means μ_1 and μ_2, the variance σ_1^2 and σ_2^2 and the correlation coefficeint ρ.

(b) Compute $E[Y|X=1]$, $E[Y|X=2]$, and the line $\mu_2 + \rho\left(\frac{\sigma_2}{\sigma_1}\right)(x-\mu_1)$.

18. Find the moment generating function, mean and variance of the gamma distribution.

19. Derive the p.d.f. of F-distribution.

20. State and prove the central limit theorem.

S–5304

Sub. Code
22MMA2E1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024.

Second Semester

Mathematics

Elective – FUZZY MATHEMATICS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a fuzzy set. Give an example.

2. Define the height of a fuzzy set. Give an example.

3. Define a dual point.

4. Write down the axiomatic skeleton for fuzzy sets intersection.

5. Define cylindric extension.

6. What do you mean by fuzzy relation equations?

7. Define a fuzzy measure.

8. Define marginal probability distributions of fuzzy measures.

9. Write a short notes on shannon entropy.

10. What is meant by information transmission?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Compute the scalar cardinality and fuzzy cardinality for the following fuzzy sets:

(i) $A = \frac{0.4}{v} + \frac{0.2}{w} + \frac{0.5}{x} + \frac{0.4}{y} + \frac{1}{z}$;

(ii) $B = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$

Or

(b) Order the fuzzy sets defined by the following membership grade functions (assuming $x \geq 0$) by the inclusion (subset) relation:

$\mu_A(x) = \frac{1}{1+20x}$, $\mu_B(x) = \left(\frac{1}{1+10x}\right)^{1/2}$,

$\mu_C(x) = \left(\frac{1}{1+10x}\right)^2$

12. (a) Show that every fuzzy complement has at most one equilibrium.

Or

(b) For all $a, b \in [0, 1]$, prove that $i(a, b) \leq \min(a, b)$.

13. (a) Let a binary fuzzy relation R be defined by the following membership matrix:

$$M_R = \begin{bmatrix} 0.7 & 0.4 & 0 \\ 0.9 & 1 & 0.4 \\ 0 & 0.7 & 1 \\ 0.7 & 0.9 & 0 \end{bmatrix}$$. Obtain its resolution form.

Or

(b) Determine the complete α – covers of the compatibility relation whose membership matrix is given below:

	1	2	3	4	5	6	7	8	9
1	1	0.8	0	0	0	0	0	0	0
2	0.8	1	0	0	0	0	0	0	0
3	0	0	1	1	0.8	0	0	0	0
4	0	0	1	1	0.8	0.7	0.5	0	0
5	0	0	0.8	0.8	1	0.7	0.5	0.7	0
6	0	0	0	0.7	0.7	1	0.4	0	0
7	0	0	0	0.5	0.5	0.4	1	0	0
8	0	0	0	0	0.7	0	0	1	0
9	0	0	0	0	0	0	0	0	1

14. (a) Let $x = \{a, b, c, d\}$. Given the basic assignment $m = (\{a, b, c\}) = 0.5$, $m(\{a, b, d\}) = 0.2$ and $m(x) = 0.3$. Find the corresponding belief and plausibility measures.

Or

(b) Prove that every possibility measure Π on $\wp(x)$ can be uniquely determined by a possibility distribution function $r : x \to [0, 1]$ via the formula $\Pi(A) = \max r(x) x \in A$ for each $A \in \wp\ (x)$.

15. (a) Show that the maximum of the measure of fuzziness defined by equation

$$f(A) = -\sum_{x \in x} (\mu_A\ (x) \log_2 \mu_A\ (x) + [1 - \mu_A(x)] \log_2 [1 - \mu_A(x)]) \text{ is } |x|$$

Or

(b) With the usual notations, prove that $H(x, y) \leq H(x) + H(y)$.

Part C

(3 × 10 = 30)

Answer any **three** questions.

16. Let the membership grade functions of sets A, B and C defined on the set $x = \{0, 1, 2, ...,10\}$ by $\mu_A(x) = \frac{x}{x+2}$, $\mu_B(x) = 2^{-x}$, $\mu_C(x) = \frac{1}{1+10(x+2)^2}$. Let $f(x) = x^2$, for all $x \in x$. Use the extension principle to derive $f(A), f(B)$ and $f(C)$.

17. (a) Write down the axiomatic skeleton for fuzzy complements. Also define the sugeno class.

    (b) Prove that $\underset{w\to\infty}{Lim}\ \min[1,(a^w + b^w)^{1/w}] = \max(a, b)$.

18. Solve the following fuzzy relation equation:

$$p0 \begin{bmatrix} 0.9 & 0.6 & 1 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{bmatrix} = [0.6 \quad 0.6 \quad 0.5]$$

19. State and prove a necessary and sufficient condition for a belief measures Bel on a finite power set to be a probability measure.

20. Prove that the function $I(N) = \log_2 N$ is the only function that satisfies $I(N.M) = I(N) + I(M)$ for all $N, M \in \mathbb{N}$ through $I(2) = 1$.

S–5305

Sub. Code
22MMA2E2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024.

Second Semester

Mathematics

Elective – NUMERICAL METHODS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Prove that Newton Raphson method has second order convergence.

2. How the constant should α be chosen to ensure the fastest possible convergence with the iteration formula $x_{n+1} = \dfrac{\alpha x_n + x_n^{-2} + 1}{\alpha + 1}$.

3. List and give the commonly used vector norms.

4. Define
   (a) Relaxation parameter
   (b) Over relaxation and
   (c) Under relaxation method

5. List the disadvantages of Quadratic splines.

6. What is a natural spline?

7. Define the order of a numerical differentiation method.

8. What is meant by Extrapolation method?

9. When does Simpson's rule give exact result?

10. What is meant by Gaussian integration method?

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Find the number of real and complex roots of the polynomial equation $P_3(x) = x^3 - 5x + 1 = 0$ using Sturm sequences.

Or

(b) Use synthetic division and perform two iterations of the Birge-Vieta method to find the smallest positive root of the equation $x^4 - 3x^3 + 3x^2 - 3x + 2 = 0$. Use the initial approximation $P_0 = 0.5$.

12. (a) Determine the condition number of the matrix $A = \begin{bmatrix} 1 & 4 & 9 \\ 4 & 9 & 16 \\ 9 & 16 & 25 \end{bmatrix}$ using the maximum absolute row sum norm.

Or

(b) For the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$

(i) Find all the eigen values and the corresponding eigen vectors.

(ii) Verify S^{-1} AS is a diagonal matrix, where S is the matrix of eigen vectors.

13. (a) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data

x	1	2	4	8
$f(x)$	3	7	21	73

Hence, estimate the values of (3) and $f(7)$

Or

(b) Given the data

x	0	1	2	3
$f(x)$	1	2	33	244

Fit quadratic splines with $M(0) = f''(0) = 0$. Hence, find an estimate of $f(2.5)$.

14. (a) The following data for the function $f(x) = x^4$ is given

x	0.4	0.6	0.8
$f(x)$	0.0256	0.1296	0.4096

Find $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation. Compare with the exact solution. Obtain the bound on the truncation errors.

Or

(b) A differentiation rule of the form $hf'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$ where $x_j = x_0 + jh$, $j = 0,1,2,3,4$ is given. Determine the values of $\alpha_0, \alpha_1, \alpha_2$ and α_3 so that the rule is exact for a polynomial of degree 4. Also find the error term.

15. (a) Find the approximate value of $\int_0^1 \frac{\sin x}{x} dx$ using

(i) mid-point rule and

(ii) two-point open type rule.

Or

(b) Determine a,b and c such that the formula $\int_0^h f(x)dx = h\left\{a\,f(0) + bf\left(\frac{h}{3}\right) + c\,f(h)\right\}$ is exact for polynomials of as high order as possible, and determine the order of the truncation error.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Obtain the complex roots of the equation $f(x) = z^3 + 1 = 0$ correct to eight decimal places. Use the initial approximation to a root as $(x_0, y_0) = (0.25, 0.25)$. Compare with the exact values of the roots $(1 + i\sqrt{3}/2$.

17. Consider the system of equations

$2x - y = 1$

$-x + 2y - z = 0$

$-y + 2z - w = 0$

$-z + 2w = 1$

(a) Set up the Gauss – Seidal iteration scheme in matrix form. Show that the scheme converges and hence find its rate of convergence.

(b) Starting with $x^{(0)} = 0$ as initial approximation, iterate three times.

18. Obtain the piecewise quadratic interpolating polynomials for the function $f(x)$ defined by the data

x	-3	-2	-1	1	3	6	7
f(x)	369	222	171	165	207	990	1779

Hence, find an approximate value of $f(-2.5)$ and $f(6.5)$.

19. For the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f'''(\xi) \;,\; x_0 < \xi < x_2 \;.$$

determine the optimal value of h, using the criteria.

(a) $|RE| = |TE|$

(b) $|RE| + |TE| = \text{minimum.}$

Using this method and the value of h obtained from the criterion $|RE| = |TE|$, determine an approximate value of $f'(2.0)$ from the following tabulated values of $f(x) = \ln x$.

x	2.0	2.01	2.02	2.06	2.12
f(x)	0.69315	0.69813	0.70310	0.72271	0.75142

Given that the maximum round-off error in function evaluation is 5×10^{-6} .

20. Find the quadrature formula $\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$, which is exact for polynomials of highest possible degree. The use the formula on $\int_0^1 \frac{dx}{\sqrt{x - x^3}}$ and compare with the exact value.

S–5307

Sub. Code
22MMA3C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

COMPLEX ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions

1. Write down the Hadamard's formula.

2. Define the following terms:

   (a) Parallel translation

   (b) Homothetic transformation

3. When will you say that the arc is said to be rectifiable?

4. Compute $\int_{|z|=1} e^z/z \, dz$.

5. Show that the function $\sin z$ have essential singularities at ∞.

6. Define a Meromorphic function.

7. State the argument principle theorem.

8. Find the poles of $cof\ z$.

9. State the Weierstrass's theorem for power series.

10. Define an entire function with an example.

Part B (5 × 5 = 25)

Answer **all** questions, choosing either (a) or (b).

11. (a) State and prove the complex form of the Cauchy-Riemann equations.

Or

(b) Find the linear transformation which carries $0, i, -i$ into $1, -1, 0$.

12. (a) Prove that the line integral $\int_{\gamma} p\,dx + q\,dy$, defined in Ω, depends only on the end points of γ if and only if there exists a function $u(x, y)$ in Ω with the partial derivatives $\frac{\partial u}{\partial x} = p, \frac{\partial u}{\partial y} = q$.

Or

(b) State and prove Cauchy's estimate and Liouville's theorem.

13. (a) State and prove the Weierstrass theorem on essential singularity.

Or

(b) State and prove the maximum principle theorem.

14. (a) State and prove the Rouche's theorem.

Or

(b) Evaluate $\int_0^\infty \frac{x \sin x}{x^2 + a^2} dx$, a real.

15. (a) State and prove the Taylor series.

Or

(b) Prove the following :

(i) $\underset{n=2}{\overset{\infty}{\pi}} \left(1 - \frac{1}{n^2}\right) = \frac{1}{2}$

(ii) $(1 + z)(1 + z^2)(1 + z^4)(1 + z^8)\ldots = \frac{1}{1 - z}$ if $|z| < 1$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. (a) State and prove the Abel's limit theorem.

(b) If $T_1(z) = \frac{z + 2}{z + 3}, T_2(z) = \frac{z}{z + 1}$, find $T_1 T_2(z)$, $T_2 T_1(z)$ and $T_1^{-1} T_2(z)$.

17. State and prove the Cauchy's theorem for a rectangle.

18. State and prove the Schwarz lemma.

19. State and prove the Residue theorem. Also find the residues of $e^z / (z - a)(z - b)$ at its poles.

20. Derive the Jensen's formula.

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S–5308

Sub. Code
22MMA3C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

TOPOLOGY – I

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A ($10 \times 2 = 20$)

Answer **all** questions.

1. Define the indiscrete topology with an example.

2. What is meant by the box topology?

3. Define a product space.

4. Define the uniform metric.

5. Whether the set Q of rational number is connected or not? Justify your answer.

6. Define the term linear continuum.

7. Prove that the real line $\mathbb{R}$ is not compact.

8. Define a Lebesgue number.

9. Define a first-countable.

10. What is meant by completely regular space?

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let X be a set and let $\mathcal{B}$ be a basis for a topology τ on X. Prove that τ equals the collection of all unions of elements of $\mathcal{B}$.

Or

(b) Prove that the collection $S = \{\pi_1^{-1}(U) / U \text{ open in } X\}$ $U = \{\pi_2^{-1}(V) / V \text{ open in } Y\}$ is a subspace for the product topology on $X \times Y$.

12. (a) Let $\{X_\alpha\}$ be an indexed family of spaces and let $A_\alpha \subset X_\alpha$ for each α. If πX_α is given either the product or the box topology, then prove that $\pi \overline{A}_\alpha = \overline{\pi A_\alpha}$.

Or

(b) State and prove the sequence lemma.

13. (a) Prove that the union of a collection of connected subspaces of X that have a point in common is connected.

Or

(b) If X is a topological space, each path component of X lies in a component of X. If X is locally path connected, the prove that the components and the path components of X are the same.

14. (a) State and prove the extreme value theorem.

Or

(b) Prove that compactness implies limit point compactness, but not conversely.

15. (a) Define a Lindelof space. Is the product of two Lindelof spaces need not be Lindelof? Justify your answer.

Or

(b) Show that a subspace of a regular space is regular.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Let X be a topological space. Prove the following conditions hold:

(i) ϕ and X are closed

(ii) Arbitrary intersections of closed sets are closed.

(iii) Finite unions of closed sets are closed.

(b) Let A be a subset of the topological space X and let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$.

17. Let X and Y be topological spaces and let $f : X \to Y$. Prove the following are equivalent:

(a) f is continuous

(b) For every subset A of X, one has $f(\overline{A}) \subset \overline{f(A)}$.

(c) For every closed set B of Y, then set $f^{-1}(B)$ is closed in X.

(d) For each $x \in X$ and each neighborhood V of $f(x)$, there is a neighborhood U of x such that $f(U) \subset V$.

18. (a) State and prove the intermediate value theorem.

(b) Let X be locally path connected. Show that every connected open set in X is path connected.

19. Prove that the product of finitely many compact spaces is compact.

20. State and prove the Urysohn metrization theorem.

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S–5309

Sub. Code
22MMA3C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

GRAPH THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define an isomorphism between two graphs with an example.

2. Show that $\delta \leq 2\varepsilon / v \leq \Delta$.

3. Define a block of a graph with an example.

4. Draw the Herschel graph.

5. Define a perfect matching. Give an example.

6. Define k-edge-chromatic graph with an example.

7. What is meant by a maximum independent set? Give an illustration.

8. Define a vertex colouring. Give an example.

9. Embed K_5 on the torus.

10. What is meant by four colour conjecture?

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Define the following terms with an illustration for each :

(i) Adjacency matrix;

(ii) Incidence matrix.

Or

(b) Prove that a graph is bipartite if and only if it contains no odd cycle.

12. (a) Prove that a connected graph has an Euler trail if and only if it has at most two vertices of odd degree.

Or

(b) If G is a simple graph with $\upsilon \geq 3$ and $\delta \geq \upsilon / 2$, then prove that G is Hamiltonian.

13. (a) State and prove the Hall's theorem.

Or

(b) If G is bipartite, then prove that $\psi' = \Delta$.

14. (a) With the usual notations, prove that $\alpha' + \beta' = \upsilon$ if $\delta > 0$.

Or

(b) Prove that in a critical graph, no vertex cut is a clique.

15. (a) (i) Prove that K_5 is non planar.

(ii) Define a plane graph with an example.

Or

(b) (i) Define the following terms with an example for each: Dual of graph and bridge of graph.

(ii) State the fine colour theorem.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. (a) Prove that in a tree, any two vertices are connected by a unique path.

(b) If G is a tree, then prove that $\varepsilon = \upsilon - 1$. Also prove every nontrivial tree has at least two vertices of degree one.

17. With the usual notations, prove that $k \leq k' \leq \delta$.

18. (a) State and prove the Berge theorem.

(b) Find the number of different perfect matching in K_{2n} and $K_{n,n}$.

19. State and prove the Brook's theorem.

20. State and prove that Euler's formula for a connected plane graph. Also if G is a simple planar graph with $\upsilon \geq 3$, then prove that $\varepsilon \leq 2\upsilon - 6$.

S–5312

Sub. Code
22MMA3E3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Third Semester

Mathematics

Elective : AUTOMATA THEORY

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. What is a transition system? When does it accept a string w?

2. Write any one property of transition function.

3. Is two grammars of different types can generate the same language? Justify your answer.

4. If the productions of G are $S \rightarrow as \mid sb \mid a \mid b$, then prove that $abab \in L(G)$.

5. Define the transpose set. Give an example.

6. If L_1 and L_2 are the subsets of $\{a,b\}^*$, prove or disprove. If $L_1 \subseteq L_2$ and L_1 is not regular, then L_2 is not regular.

7. Describe the set $\{\wedge, 0, 00, 000, \ldots\}$ by regular expression.

8. Write any two application of pumping lemma.

9. Give an illustration for a derivation tree.

10. Let G be a grammar $S \to Sbs \mid a$. Is G ambiguous? Justify.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Draw a block diagram of a finite automation. Also explain the various components of it.

Or

(b) Consider the finite state machine whose transition function δ is given in the following table :

	Inputs	
$\rightarrow$ State	0	1
(q_0)	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Take $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0,1\}$ and $F = \{q_0\}$. Give the entire sequence of states for the input string 110101.

12. (a) Find a grammar generating $L = \{a^n b^n c^i / n \geq 1, i \geq 0\}$.

Or

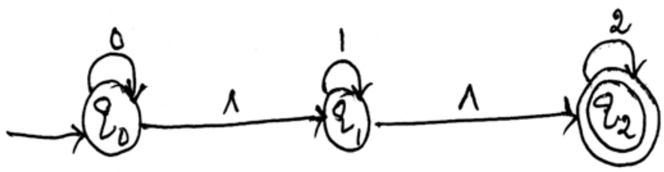
(b) Show that the set of all non-palindromes over $\{a, b\}$ is a context-free language.

13. (a) Consider the grammar G given by $S \to 0SA_1 2$, $S \to 012$, $2A_1 \to A_1 2$, $1A_1 \to 11$. Test whether $001122 \in L(G)$.

Or

(b) Let Z_0 denote the family of type of languages. Prove that the class Z_0 is closed under concatenation.

14. (a) Consider a finite automation with $\wedge$-moves, given the following figure. Obtain an equivalent automation without $\wedge$-moves.



Or

(b) With the usual notations, prove that $(a+b)^* = a^*(ba^*)^*$.

15. (a) If $A \overset{*}{\Rightarrow} W$ in G, then prove that there is a leftmost deviation of W.

Or

(b) Find a grammar in CNF equivalent to $S \to aAbB$, $A \to aA / b$, $B \to bB / b$.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. If L is the set accepted by NDFA, then prove that there exists a DFA which also accepts L.

17. Show that every monotonic grammar G is equivalent to a type 1 grammar.

18. Construct context-free grammars to generate the following :

    (a) $\{0^m 1^n \mid 1 \leq m \leq n\}$

    (b) The set of all strings over $\{0,1\}$ containing twice as many 0's and 1's.

19. (a) Show that $L = \{0^i 1^i \mid i \geq 1\}$ is not regular.

    (b) If L is a regular set over Σ, then prove that $\Sigma^* - L$ is also regular over Σ.

20. Let G be $S \to AB$, $A \to a$, $B \to C / b$, $C \to D$, $D \to E$ and $E \to a$. Eliminate unit productions and get an equivalent grammar.

S–5314

Sub. Code
22MMA4C1

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a normed space. Give an example.

2. Define a bounded linear map.

3. What is meant by convex body?

4. Define a convex body.

5. Define a Bomach space with an example.

6. Define the standard Schauder basis.

7. Define the graph of F.

8. Define a projection.

9. Define an orthonormal basis.

10. State projection theorem.

Part B (5 × 5 = 25)

Answer **all** questions choosing either (a) or (b).

11. (a) Define a normed space with an example. Let *Y* be a subspace of a normed space X. Prove that Y and its closure $\overline{Y}$ are normed spaces with the induces norm.

Or

(b) Let *X* and *Y* be normed spaces and $F : X \to Y$ be a linear map such that the range *R(F)* of *F* is finite dimensional. Prove that F is continuous if and only if the zero space *Z (F)* of F is closed in *X*.

12. (a) State and prove Hahn–Banach separation theorem.

Or

(b) State and prove Hahn–Banach extension theorem.

13. (a) Prove that a normed space *X* is a Banach space if and only if every absolutely summable series of elements in *X* is summable in *X*.

Or

(b) State and prove uniform boundedness principle.

14. (a) Let *X* be a linear space over *K*. Consider subsets *U* and *V* of *X*, and $k \in K$ such that $U \subset U + kU$. Prove that for every $x \in U$, there is a sequence (v_n) in *V* such that $x - \left(v_1 + kv_2 + ... + k^{n-1}v_n\right) \in k^n U$, $n = 1$, 2, ...

Or

(b) Let *X* and *Y* be normed spaces. Let $F : X \to Y$ be a linear map such that the subspace *Z(F)* is closed in *X*. Define $\widetilde{F} : \dfrac{X}{Z(F)} \to Y$ by $\widetilde{F}(x + Z(F)) = F(x)$ for $x \in X$. Prove that F is an open map if and only if F is an open map.

15. (a) Derive the Bessel's inequality.

Or

(b) State and prove the Parseval formula.

Part C $(3 \times 10 = 30)$

Answer any **three** questions.

16. Let X be a normed space. Prove the following conditions are equivalent:

(a) Every closed and bounded subset of X is compact.

(b) The subset $\{x \in X : \|x\| \leq 1\}$ of X is compact.

(c) X is finite dimensional.

17. State and prove the Taylor – Foguel theorem.

18. State and prove the uniform bounded principle.

19. Let X arid Y be Banach spaces and $F : X \to Y$ be a closed linear map. Prove that F is continuous.

20. State and prove the Riesz representation theorem.

S–5315

Sub. Code
22MMA4C2

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024.

Fourth Semester

Mathematics

TOPOLOGY – II

(CBCS – 2022 onwards)

Time : 3 Hours   Maximum : 75 Marks

Part A   ($10 \times 2 = 20$)

Answer **all** the questions.

1. Define the one-point compactification.

2. Is the rationals Q locally compact? Justify your answer.

3. What is meant by completely regular space?

4. When will you say that two compactification is said to be equivalent?

5. Define countable locally finite.

6. What is meant by locally discrete?

7. When will you say that the metric space is said to be complete?

8. Define an equicontinuous.

9. What is meant by compact open topology?

10. Define a Baire space. Give an example.

Part B $(5 \times 5 = 25)$

Answer **all** questions choosing either (a) or (b).

11. (a) Let X be a Housdorff space. Prove that x is locally compact if and only if given x in x and given neighborhood U of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subset U$.

Or

(b) Let x be space and let $\mathcal{D}$ be a collection of subsets of X that is maximal with respect to the finite intersection property. Prove that any finite intersection of elements of $\mathcal{D}$ is and element of $\mathcal{D}$.

12. (a) Show that a product of completely regular spaces is completely regular.

Or

(b) Let $A \subset X$ and let $f : A \to Z$ be a continuous map of A into the Hausdorff space Z. Prove that there is at most one extension of f to a continuous function $g : \overline{A} \to Z$.

13. (a) Let A be a locally finite collection of subset of X. Prove that the collection $\mathcal{B} = \{\overline{A}\}_{A \in \mathcal{A}}$ of the closures of the elements of $\mathcal{A}$ is locally finite.

Or

(b) Let x be normal and let A be closed G_s set in X. Prove that there is a continuous function $f : x \to [0, 1]$ such that $f(x) = 0$ for $x \in A$ and $f(x) > 0$ for $x \notin A$.

14. (a) Define a complete metric space with an example. Also prove that there is a metric for the product space $\mathbb{R}^{\omega}$ relative to which $\mathbb{R}^{\omega}$ is complete.

Or

(b) If X is locally compact, or it X satisfies the first countability axiom, then prove that X is compactly generated.

15. (a) If Y is locally compact Hausdorff, then prove that composition of maps $\mathcal{C}(x,y) \times \mathcal{C}(y,z) \rightarrow \mathcal{C}(x,z)$ is continuous.

Or

(b) Show that every locally compact Hausdorff space is Baire space.

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Prove that an arbitrary product of compact spaces is compact in the product topology.

17. Let X be a completely regular space. If Y_1 and Y_2 are two compactification of X satisfying the extension property, then prove that Y_1 and Y_2 are equivalent.

18. Let X be a regular space with a basis $\mathcal{B}$ that is countably locally finite. Prove that X is normal and every closed set in x is a G_S set in x.

19. Prove that a metric space (x,d) is compact if and only if it is complete and totally bounded.

20. State and prove the Baire category theorem.

S–5316

Sub. Code
22MMA4C3

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024

Fourth Semester

Mathematics

OPERATIONS RESEARCH

(CBCS – 2022 onwards)

Time : 3 Hours Maximum : 75 Marks

Part A $(10 \times 2 = 20)$

Answer **all** questions.

1. Define a connected network.

2. Define a cut and the cut capacity of a network.

3. Define the following terms:

   (a) Forward Pass

   (b) Backward Pass

4. Define the pessimistic time estimate.

5. Define the following terms:

   (a) Purchasing cost

   (b) Setup cost

6. Define the effective lead time.

7. Identify the customer and the server for the following situations:

   (a) Planes arriving at an airport

   (b) Check-out operation in a supermarket

8. What is forgetfulness property?

9. Draw the transition-rate diagram.

10. Write down the formula for L_s and L_q of the model $(M / M / C):(G D / \infty / \infty)$.

Part B $(5 \times 5 = 25)$

Answer **all** questions, choosing either (a) or (b).

11. (a) Narrate the minimal spanning tree algorithm.

Or

   (b) Explain three-Jug puzzle with a suitable diagram.

12. (a) Construct the network diagram comprising activities $B, C, \ldots\ldots Q$ and N such that the following constraints are satisfied.

   $B < E, F; C < G, L; E, G < H; L, H < I; L < M; H < N, H < J; I, J < P; P < Q$. The rotation $X < Y$ means that the activity X must be finished before Y can begin.

Or

   (b) Explain the critical path computations.

13. (a) A company stocks an item that is consumed at the rate of 50 units per day. It costs the company \$20 each time an order is placed. An inventory unit held in stock for a week will cost \$.35.

(i) Determine the optimum inventory policy, assuming a lead time of 1 week.

(ii) Determine the optimum number of orders per year (Based on 365 days per year).

Or

(b) Find the optimum order quantity for a product for which the price break are as follows:

Quantity	Unit Cost (Rs.)
$0 \le y_1 < 500$	10.00
$500 \le y_2$	9.25

The monthly demand for the product is 200 units, the cost of storage is 2% of the unit cost and the cost of ordering is Rs.350.00.

14. (a) Explain the role of exponential distribution.

Or

(b) In a bank operation, the arrival rate is 2 customers per minute. Determine the following:

(i) The average number of arrivals during 5 minutes.

(ii) The probability that no arrivals will occur during the next 0.5 minute.

(iii) The probability that at least one arrival will occur during the next 0.5 minute.

(iv) The probability that the time between two successive arrivals is at least 3 minutes.

15. (a) Describe the notation $(a/b/c):(d/e/f)$.

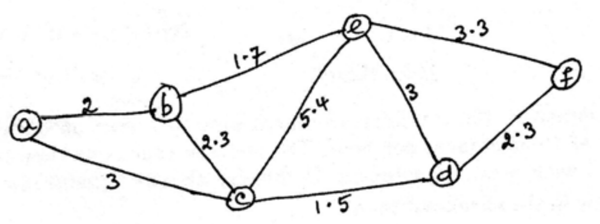
Or

(b) A petrol station has two pumps. The service time follows the exponential distribution with mean 4 minutes and cars arrive for service in a Poisson process at the rate of 10 cars per hour. Find the probability that, a customer has to wait for service. What proportion of time the pumps remain idle?

Part C (3 × 10 = 30)

Answer any **three** questions.

16. Use Dijkastra's algorithm to find the shortest path from source 'a' to destination *f* from the following network.



17. A small project is composed of activities whose time estimates are listed in the table below : Activities are identified by their beginning (i) and ending (j) node numbers.

Activity	Estimated Duration (Weeks)		
i – j	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

(a) Draw the project network.

(b) Find the expected duration and variance for each activity. What is the expected project length?

(c) Calculate the variance and standard deviation of the project length. What is the probability that the project will be completed?

(i) At least 4 weeks earlier than expected?

(ii) No more than 4 weeks later than expected time?

(d) If the project due is 19 in weeks, what is the probability of meeting the due date? Given:

Z	0.5	0.67	1.00	1.33	2.00
p	0.1915	0.2486	0.3413	0.4082	0.4772

18. Neon lights on the U of A compus are replaced at the rate of 100 units per day. The physical plant orders the neon lights periodically. It costs $100 to initiate a purchase order. A neon light kept in storage is estimated to cost about $0.02 per day. The lead time between placing and receiving an order is 12 days. Determine the optimal inventory policy for ordering the neon lights.

19. Explain in details about the relationship between the exponential and poisson distributions.

20. B and K Groceries operates with three check out counters. The manager uses the following schedule to determine the number of counters in operation, depending on the number of customers in store.

No. of customers in store	No. of customers in operation
1 to 3	1
4 to 6	2
More than 6	3

Customer arrive in the counters area according to a Poisson distribution with a mean rate of 10 customers per hour. The average check-out time per customer is exponential with mean 12 minutes. Determine the steady-state probability p_n of n customers in the check-out area.

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